

Pattern recognition computation using action potential timing for stimulus representation

J. J. Hopfield

Division of Chemistry and Chemical Engineering, and Division of Biology, California Institute of Technology, Pasadena, California 91125, USA

A computational model is described in which the sizes of variables are represented by the explicit times at which action potentials occur, rather than by the more usual 'firing rate' of neurons. The comparison of patterns over sets of analogue variables is done by a network using different delays for different information paths. This mode of computation explains how one scheme of neuroarchitecture can be used for very different sensory modalities and seemingly different computations. The oscillations and anatomy of the mammalian olfactory systems have a simple interpretation in terms of this representation, and relate to processing in the auditory system. Single-electrode recording would not detect such neural computing. Recognition 'units' in this style respond more like radial basis function units than elementary sigmoid units.

HERE I describe a recognition problem that occurs in many guises and sensory modalities, and that can be efficiently solved when analogue information is encoded (or represented) using action potential timing. I emphasize the importance of neurobiological information representations for their ease in computing useful results, rather than viewing the choice of representation as chiefly a means efficiently to transmit 'information' as defined in communication theory¹⁻³.

The choice of information representation is of vital importance in making a computation easy, as is illustrated by the question 'is 3630225 divisible by 7?'. If the number 3630225 is in base 10, answering this takes a bit of work. If the number 3630225 is in base 7, the answer is immediate from inspection of the last digit. The choice of representation for analogue information is equally important for neural computation.

In this paper, analogue information is represented by using the timing of action potentials with respect to an ongoing collective oscillatory pattern of activity. This is a long-hypothesized representation⁴ for which there is now appreciable evidence⁵ for special cases. Experiments⁶ on rat hippocampal 'place cells' indicate that the phase of neural activity with respect to the ongoing local theta-rhythm correlates with the location of the rat within that receptive field. Action potential phase-time coding is also found in electric fish⁷. The particular representation of analogue information used here is created by elementary cellular biophysics in a neuron having a subthreshold oscillatory membrane potential. In contrast, most previous descriptions of neurobiological computation are based on either firing-rate coding or place-cell coding of analogue information.

The scheme computes in this analogue representation by combining information through pathways with different delays. This mode of computation is believed to be responsible for the movement-sensitive visual neurons⁸, and for spatial localization of sounds in the barn owl⁹. The analysis requires a coherence of the oscillation across a localized set of neurons, but such coherence is common⁵ at frequencies ranging from 1 to 100 Hz.

Thus the major basis phenomena needed in this new analysis are all known to occur in neurobiology. These known components, working together, can rapidly and efficiently perform computations that are essential to pattern recognition, and that are much more difficult to perform in a rate-coding framework.

I analyse the problem of 'analogue pattern recognition'. This problem, in slightly different guises, occurs in the recognition of

colours, visual patterns, odours and sound quality, and is a very general task which neurobiology solves rapidly. Surprisingly, this problem is not easily solved by most rate-code neural models.

Calculating analogue match

When a stellar constellation is pictured on the cover of a magazine we ask ourselves whether it is Orion or Ursa Major? Is a faint smell from a recently opened bottle of Bordeaux? These disparate questions are examples of analogue match problems. The basic stimulus is a pattern of analogue values, the ratios and scale of which define the stimulus quality and intensity. For example, the ratios of the lengths of the lines connecting the different pairs of stars determine the shape of the constellation, while the length scale of the lines describes the size of the photographic rendering. In the olfactory example, the currents generated in the individual neurons (or classes of neurons) are the analogue pattern. Sensory neurons in the general olfactory system are not highly specific, and a particular odour must be identified by its pattern of strength of excitation over the ensemble of primary sensory neurons¹⁰.

The computational problem can be posed as follows. Let a nervous system have implicit knowledge of several stimuli a, b, c, \dots , each one characterized by a list of numbers $X_a = \{X_{a,1}, X_{a,2}, \dots, X_{a,i}, \dots\}$. An unknown stimulus is presented which has the properties X_u . We wish to determine for which (if any) of a, b, c can it be considered that

$$X_u = \sim \lambda X_a \quad \text{or} \quad X_{u,i} = \sim \lambda X_{a,i} \quad \text{for all } i \quad (1)$$

and also the value of λ . The form (1) implies scale-invariant recognition of a stimulus quality and knowledge of its intensity or size.

A simple conventional neural model has difficulties in solving this problem. Consider a single 'grandmother cell' neuron which is to recognize a single odour, and distinguish it from other, perhaps unknown, odours. The neuron has a monotonic input-output relationship such as

$$V_{\text{out}} = 1/(1 + e^{-v_{\text{in}}}); \quad v_{\text{in}} = v_{\text{bias}} + \sum W_i X_{u,i} \quad (2)$$

The W_i are the 'synaptic weights' of the problem; $V_{\text{out}} > 0.5$ might be taken as recognition of the odour¹¹.

This elementary system is not satisfactory because any odour u for which the scalar product $W \cdot X_u$ is positive will, if strong enough, be recognized as the known odour. A system which separates odour quality (which involves only the relative components of X) and odour intensity, and judges the match of odour quality independently of the scale factor, is essential. Preprocessing the input X through a network which converts X into a vector of fixed length is one way to solve the problem. Conceptually, this requires the computation of the euclidian length of the raw input vector, and then division of all inputs by that factor. This mathematics is not natural to elementary neural circuits.

This structure of network, even with normalization, has two additional undesirable features. First, sensitivity to minor components is lost. If the known odour had intensity ratios 5:5:1 for three variables, the optimal weights to recognize that odour correctly are also in the ratio 5:5:1, so the third component is multiplied by a small weight, and is not a substantial contributor to v_{in} even if it is of great significance.

An additional problem is involved if we attempt to split the recognition problem into sub-parts. If the raw inputs are split into two groups, and each is processed by separate networks of this type, the outputs of the two 'grandmother units' cannot simply be added to gain confidence in a recognition. For if the two units both seem to recognize the odour, but the scale factors described by the two normalizing processes are very different, the overall pattern is not a good match.

The encoding and computation model described in the next sections uses action potential timing to carry information. It further differs from the above (1) in solving analogue pattern recognition problems simply and naturally; (2) in allowing problems to be broken into smaller sub-parts and the results then combined; and (3) in embedding the information about the pattern to be recognized in a pattern of axonal or cellular time delays. Synaptic strengths determine only the importance of particular variables, not the pattern to be recognized, and therefore require little precision. The processing is intrinsically rapid.

Time-encoding information

The model involves 'encoding neurons' which produce action potentials, and which are also influenced by an oscillatory drive. (In the vertebrate olfactory system, these units might correspond to the mitral cells of the olfactory bulb.) In the model, a cell will produce an action potential at time t if the cell potential $u(t) > u_{\text{thresh}}$ and if no action potential has been generated more recently than the refractory period τ_r . It is not necessary to consider the detailed biophysics of action potentials.

The other essential feature of the model is an oscillatory subthreshold variation of the membrane potential for the encoding neurons. Such an oscillatory potential can be generated by intrinsic cellular effects^{12,13} or multineuron circuitry¹⁴. The encoding cells are also presumed to have a time constant which is somewhat shorter than the oscillation period. The cell potential of encoding cell j (except for the rapid action potential component) will thus be taken to be

$$(u_j(t) - u_{\text{thresh}}) / R = I_j(t) - I_0 - A(1 - \cos 2\pi ft) \quad (3)$$

where A and I_0 are positive constants, R is the cell resistance, $I_j(t)$ is the input current to cell j , and f is the frequency of subthreshold oscillation. For simplicity, $I_j(t)$ is assumed to be excitatory, but inhibitory schemes can also be developed. The cosine function is used for illustration; in general, the periodic oscillation will have a more complex form. In the absence of an input current $I_j(t)$, the cell exhibits only subthreshold membrane potential oscillations. When the input current exceeds I_0 , action potentials will be generated. The dead time and level of input can together prevent the cell from generating two action poten-

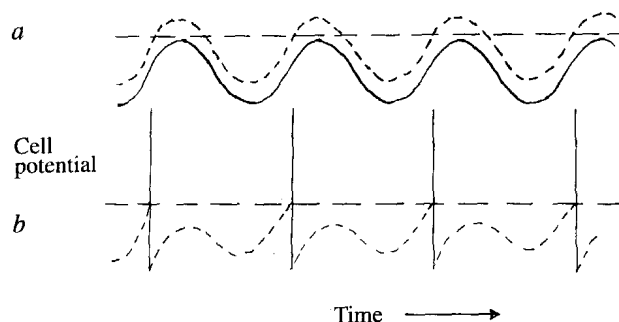


FIG. 1 a, When there is no input current, the subthreshold cell potential oscillation (solid line) never exceeds the firing threshold (horizontal broken line). When an input current is added, the cell potential (neglecting currents which flow during an action potential) will cross threshold, as shown by the broken curve. b, The cell potential corresponding to the broken curve in a, including the effect of the currents which flow during action potentials, which leave the cell in a depolarized state from which it slowly recovers. The cell potential never again exceeds the threshold for spike generation until the next cycle of the periodic oscillation. Thus the cell fires only on the upward threshold crossings of the broken line in a.

tials within one period of the oscillation (Fig. 1). The way several cells respond to an analogue input pattern is shown in Fig. 2. The time τ_j each neuron j fires in advance of the maximum of the subthreshold oscillation is determined by its input current, thus encoding the analogue information I_j as a time advance τ_j . The firing frequency for all active neurons is simply f .

The functional form of the time advance as a function of the input current is needed for further analysis. This is described by a function $\tau(I)$, which is determined by the shape of the oscillatory part of the cell potential and any nonlinear transformation which may have been performed on the inputs (such as the logarithmic intensity transformation in the visual system¹⁵). I assume in the following section that the preprocessing of the fundamental sensory variable X has been matched to the shape of the top of the oscillatory cycle to yield a net encoding such that signal input channel j has an advance

$$\tau_j = \sim \ln(X_j / \delta + 1) = \sim \ln(X_j / \delta) \quad \text{for strong odours} \quad (4)$$

where δ is a constant scale factor.

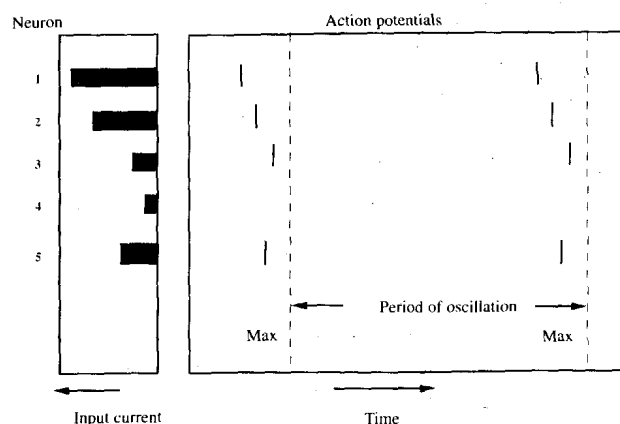


FIG. 2 The action potential of five different neurons driven with different input currents (analogue signal strengths). Neuron 4 produces no action potentials because the input current is insufficient to ever drive that cell above threshold. The other neurons all fire once during each oscillation period, but the time τ_j each neuron j fires in advance of the maximum of the subthreshold oscillation is determined by its input current, and is larger for neurons driven with a greater input current. The firing frequency remains f over a considerable range of input current if the refractory period is long.

It has so far been presumed that a neuron never fires more than once during a cycle, which will be the case within a limited regime of input current, frequency, postspike depolarization and cell time constant. Outside this regime, the first action potential will occur at the location described, but additional action potentials may also be generated within a period. In this case, the first spike will carry appropriate analogue information on its timing, and analogue information will also be carried in the number of action potentials which occur within a cycle. Thus more than one representation of analogue information can be simultaneously present. Rat hippocampal cells⁶ seem to respond in such a dual fashion.

Decoding and recognition

When a cell has a relatively short integration time constant, its response to action potentials arriving from different afferents is sensitive to the relative times of arrival of the action potentials. A cell can thus serve as a 'coincidence detector' for action potentials impinging on it by different pathways. The auditory place cells found in the barn owl⁹ are believed to 'compute' azimuthal position of a sound source by detecting a time coincidence between signals which appear delayed with respect to each other by the different lengths of the paths between the sound source and each ear, and then have compensating axonal delays which result in a coincidence of arrival on a target cell. The decoding or recognition scheme to be described is a generalization of this idea.

More generally, when a pattern consists of a set of features occurring in a given time relationship it can be recognized by a processing system which uses time delays¹⁶ and coincidence detection. These time delays are organized so that the features recognized by feature-detecting neurons, although occurring at different times, produce signals which arrive simultaneously at a recognition neuron which then responds maximally. This motif is found in visual systems that detect the directional movement of edges⁸. This delay organization is believed to be how the moustache bat processes FM sonar echoes¹⁷. The biophysical mechanism of delay can be synaptic, axonal or cellular. Time-delay decoding is a powerful engineering tool, and has been successfully used in recognizing words in continuous natural speech¹⁸.

In biosonar an animal generates a scanning sound, and the reflected sound has two kinds of information. The relative times of the received sound features describe the object being detected, whereas the overall time of the return (with respect to the scanning sound) represents a totally different but significant variable, the distance to the reflecting object.

In the examples cited above, the time pattern to be decoded originates with the nature of the time-dependent stimulus. By contrast, the time pattern to be decoded in the present case originates with the encoding scheme itself, and is created even for a constant input stimulus. However, a pattern in time can be decoded by a network of coordinated time delays, regardless of its origin.

Logarithmic encoding (see equation (4)) is particularly useful, because it makes the relative timing of the action potentials encoding a pattern independent of the intensity or scale. If the strength of an odour is increased by a factor of 2, each action potential will be advanced in time by the same amount, and the (time-shifted) pattern will be recognized in a scale-invariant fashion by a time-delay network. The recognition event for the stronger case will simply occur earlier with respect to the maxima of the oscillatory potential. Information about the scale of a recognized pattern is thus encoded in the time at which the recognition event occurs in a 'grandmother cell' or network. A recognition can be readily broken into two parts and recombined as follows. The inputs are divided into two groups which are independently coded and recognized by time-delay networks. If these two independent recognitions correspond to the same intensity, they agree and support each other. If they correspond

to different intensities, then the overall pattern is wrong. However, if they correspond to the same intensity then the two recognition neurons will generate action potentials at the same time; otherwise they will not. Coincidence detection is all that is needed to recombine the parts and to see if the two partial results support each other.

If input current levels are so high that multiple spikes are present in a single cycle, the correct recognition will still be made. However, in addition, spurious recognitions due to these other spikes might sometimes be made later in the cycle, but later recognitions might be discriminated against. Alternatively, synaptic suppression with an appropriate recovery time could chiefly suppress the effect of multiple spikes within a single cycle. Multiple spikes within a single cycle will therefore not debilitate this mode of computation.

Comparison with mammalian olfaction

The three features necessary to use this representation for recognizing odours are present in the mammalian olfactory system. The olfactory bulb, which is the earliest processing area, shows pronounced global oscillations at about 40 Hz¹⁹. Thus the mitral cells of the olfactory bulb, which receive inputs directly from sensory cells and send axons directly to the piriform cortex²⁰, could encode the input current by the mechanism described. The pyramidal cells in the piriform cortex receive inputs from the axon tract from the olfactory bulb, and from the association fibres within the piriform cortex itself. The anatomy and very slow propagation velocities generate a distribution of time delays as long as 20 ms across the rat piriform cortex²¹. This anatomy is well suited to decoding the kind of analogue coding we have suggested, but makes no sense if the information arriving is primarily rate coded.

The response of principal neurons in the olfactory bulb to the presence of odours is unexpectedly complex²². In an elementary model of odour processing, a mitral cell in the olfactory bulb might be expected to respond strongly to one odour and weakly to another, amplifying the discrimination between odours which is begun by the broadly tuned sensory neurons. Although some selectivity is seen, the response of many mitral cells also strongly depends on concentration and on the timing within the breathing and oscillation cycle. This mitral cell response may be an indication of a very different way of encoding analogue information.

Oscillation occurs in other olfactory systems as well. The locust²³ antenna ganglion shows coherent oscillation at about 20 Hz, whereas the procerebral lobe of the slug²⁴ shows field potential oscillations at about 0.5 Hz. Many reasons for the oscillation in olfaction have been suggested, ranging from an epiphenomenon of the circuitry to explicit uses such as enhancing differences between signals²⁵. The scheme I outline here is different from these in that the function of the oscillation, to recode analogue information into the time domain, cannot readily be done by more conventional means.

Discussion

The model described here is a particular example of a general idea, using action potential timing to encode analogue information, and using time-delay networks to compute in this representation. Such a coding system has great computational power and speed compared with a network using an 'instantaneous rate' coding of variables. A calculation of analogue pattern match can be done in 25 ms with neurons that have firing rates of no more than 40 Hz.

The cell model, the coding and the decoding networks are overly simple and without feedback pathways, so a correspondence between the described model and electrophysiology cannot be completely direct. However, the viewpoint does explain many of the enigmatic features of olfactory systems.

The use of such computation could be widespread. Much of the palaeocortex shares the architecture of the piriform cortex²⁶,

and if piriform cortex computes through time-delay representations, so might other parts of the palaeocortex. The frontal cortex in higher mammals, where cells typically have very low firing rates, is another obvious candidate for the use of timing as a means of representing information.

Elaboration on this basic theme could involve more complex subthreshold behaviour, or neurons with more complicated ways of transforming analogue variables to time sequences, and multiple action potentials during a single oscillation cycle. I have described only the simplest of possible encodings. The important general point is the combination of speed and ease of computation with a time-domain representation of analogue variables, and the different point of view such ideas bring to experiments. Understanding how neurobiology functions requires an understanding of both how information is encoded and how that encoded information can be used in a subsequent computation (decoded)⁵.

Single-electrode recording is completely incapable of elucidating the nature of this new representation/computation model. Because weakly driven cells do not respond at all, and strongly driven cells all fire at the same frequency, conventional electrophysiology would simply conclude that the cells are broadly tuned. The existence of such a simple and neurobiologically plausible alternative computational model underlines the potential importance of multielectrode experiments.

Synchronized action potentials play a very different role in the processing described here from that which they have recently been hypothesized to play in the visual system^{27,28}. If information is encoded as hypothesized, then neurons that receive little input fire with very little time advance (or none at all), and will appear synchronized. These are generally the neurons with the least useful information about an odour. The neurons representing the strong components of the odour fire earlier in time and will not appear synchronous.

The dynamic range and accuracy of pattern recognition in such a scheme depend on the time resolution available. Azimuthal binaural sound localization by owls has an angular accuracy of 1°, corresponding to a time delay of 20 µs²⁹. The 25-ms characteristic time of a 40 Hz oscillation leaves a lot of room for resolution and dynamic range if the neural architecture has been optimized for time-delay processing. If the 5 Hz theta-rhythm is used for encoding, slower synapses would be adequate for decoding. This representation of analogue information does not require synapse strengths of great precision, because the stored analogue patterns against which the incoming information is to be compared are contained in the time delays. The

strength of a synapse reflects the importance given to a particular input variable, not the size of a variable.

When sensory signals themselves have time-dependent structure, the analogue value to time-delay computations can be used without the need for an auxiliary oscillatory pattern. Binaural sound localization in the bat shows some indications of computation of intensity to time delay. Binaural cells in the medial superior olive are optimally tuned both in time delay between the two ears and in an intensity difference, with greater relative sensitivity to intensity difference. Harnischfeger *et al.*³⁰ note that "for usual latency reasons, and the way sound sources should behave (shadowing) time delay and intensity go hand-in-hand". That is, a network that can work with binaural time delay for azimuthal localization of sound would naturally also function on the basis of intensity differences, because elementary cellular biophysics results directly in time delays resulting from stimulus intensity differences. This system can therefore also be understood to use intensity-to-time-delay encoding and processing, except that in this case an oscillatory potential is not needed. Intensity-to-time encoding in the olfactory system might also be used without oscillations in conjunction with exploratory behaviour such as sniffing or antenna waving, which could serve as the onset for intensity-to-time encoding on a different timescale and at the level of the sensory cells. In this regard it is interesting to note that olfactory sensory cells in the frog have different time responses for different stimuli³¹.

If this kind of computation takes place in neurobiology, the time delays appropriate to a new stimulus must be learned. The simplest scheme is to have a multiplicity of time delays, direct or indirect, available via different synapses. This would be easily done in a structure like the piriform cortex, where the recurrent collaterals allow many possible indirect paths of different time delay between two cells. In such a structure the correct time delays would merely need to be selected by strengthening appropriate synapses using a Hebb-like rule. Alternatively, when synaptic delays are important these delays might be modified when the pre- and postsynaptic cells fired in near synchrony.

Computer scientists have noted that 'radial basis function units', the response of which is maximal for the chosen template pattern, are often better for engineering pattern-recognition networks than are the simple sigmoid response units^{32,33} so often used in modelling. The time-delay encoding of analogue patterns results in the units of a time-delay decoding network also having maximal response for a template analogue pattern. Thus neurophysiology can generate this powerful radial basis-function behaviour in a simple feedforward network. □

Received 4 October 1994; accepted 18 May 1995.

- McClurkin, J. W., Optician, L. M., Richmond, B. J. & Gawne, T. J. *Science* **253**, 675–677 (1991).
- Bialek, W. & Rieke, F. *Trends Neurosci.* **15**, 428–434 (1992).
- Atick, J. J. *Network* **3**, 213–251 (1992).
- Perkel, D. H. & Bullock, T. H. *Neurosci. Res. Prog. Bull.* **6**, 221–248 (1968).
- Bullock, T. H. A. *Rev. Neurosci.* **16**, 1–15 (1993).
- Burgess, N., O'Keefe, J. M. & Recce, M. in *Advances in Neural Information Processing Systems* Vol. 5 (eds Hanson, S. J., Giles, C. L. & Cowan, J. D.) 929–936 (Morgan Kaufman, San Mateo, CA, 1993).
- Heiligenberg, W. F. *Neural Nets in Electric Fish* 51–60 (MIT Press, Cambridge, MA, 1991).
- Reichardt, W. Z. *Naturf.* **12b**, 448–457 (1957).
- Carr, C. E. & Konishi, K. J. *Neurosci.* **10**, 3227–3246 (1990).
- Kauer, J. S. *Trends Neurosci.* **14**, 79–85 (1991).
- Herz, J., Krogh, A. & Palmer, R. G. *Introduction to the Theory of Neural Computation* (Addison Wesley, Redwood City, CA, 1991).
- Alonso, A. & Llinas, R. R. *Nature* **342**, 175–177 (1989).
- Silva, L. R., Amital, Y. & Connors, B. W. *Science* **252**, 432–435 (1991).
- Shepherd, G. M. & Brayton, R. K. *Brain Res.* **175**, 377–382 (1979).
- Rodieck, R. W. *The Vertebrate Retina* (Freeman, San Francisco, 1973).
- Tank, D. W. & Hopfield, J. J. *Proc. natn. Acad. Sci. U.S.A.* **84**, 1896–1900 (1987).
- Kuwabara, N. & Suga, N. J. *Neurophysiol.* **69**, 1713–1724 (1993).
- Unnikrishnan, K. P., Hopfield, J. J. & Tank, D. W. *IEEE Trans. Signal. Process.* **39**, 698–713 (1991).

- Adrian, E. D. *J. Physiol., Lond.* **100**, 459–473 (1941).
- Shepherd, G. M. *The Synaptic Organization of the Brain* 152–183 (Oxford Univ. Press, Oxford, 1979).
- Haberly, L. *Chem. Senses* **10**, 219–238 (1985).
- Nickell, W. T. & Shipley, M. T. in *Science of Olfaction* (eds Serby, M. J. & Chobor, K. L.) 172–212 (Springer, New York, 1992).
- Laurent, G. & Naraghi, M. J. *Neurosci.* **14**, 2993–3004 (1994).
- Delaney, K. R. *et al. Proc. natn. Acad. Sci. U.S.A.* **91**, 669–673 (1994).
- Tank, D. W., Gelperin, A. & Kleinfeld, D. *Science* **265**, 1819–1820 (1994).
- Shepherd, G. M. *The Synaptic Organization of the Brain* 289–307 (Oxford Univ. Press, Oxford, 1979).
- Eckhorn, R. *et al. Biol. Cybern.* **60**, 121–130 (1988).
- Gray, C. M. & Singer, W. *Proc. natn. Acad. Sci. U.S.A.* **86**, 1698–1702 (1989).
- Carr, C. E. & Konishi, M. *Proc. natn. Acad. Sci. U.S.A.* **85**, 8311–8315 (1988).
- Harnischfeger, G., Neuweiler, G. & Schlegel, P. J. *Neurophysiol.* **53**, 89–109 (1985).
- Reviai, M. F., Sicard, G., Duchamp, A. & Holley, A. *Chem. Senses* **7**, 175–190 (1982).
- Girosi, F. & Poggio, T. *Biol. Cybern.* **63**, 169–176 (1990).
- Hossain, O. & Fahmy, M. M. *Neural Computat.* **6**, 927–943 (1994).

ACKNOWLEDGEMENTS. I thank A. V. M. Herz, J. F. Hopfield, G. Laurent, D. W. Tank and M. C. Waltham for criticism of the manuscript, and I. Aleksander for the hospitality of Imperial College, where part of this work was done. This work was supported by the NSF and the Ron and Maxine Linde Venture Fund.