

1. Consider a quantum algorithm that starts with an initial state  $|\psi_0\rangle$ , to which the sequence of unitary gates  $U_1, U_2, \dots, U_T$  is to be applied. Suppose that only approximations  $V_i$  to the gates  $U_i$ , such that  $\|U_i - V_i\|_2 \leq \delta$ , are available to us. How far (in  $\ell_2$ -norm) is the final state  $|\xi_T\rangle = V_T V_{T-1} \cdots V_1 |\psi_0\rangle$ , which we get from the approximate computation, from the ideal state  $|\psi_T\rangle = U_T U_{T-1} \cdots U_1 |\psi_0\rangle$ ? Thus, how accurate do we require our “real” gates to be, for a polynomial time computation?

**Hint:** Apply the hybrid argument that we used to show a lower bound for the search problem.

2. Show that a quantum error correcting code that is resilient against both a bit- and a phase-flip on one qubit is also resilient against *any* linear error on one qubit. Verify this for the 9-qubit Shor code.

What about linear errors on more qubits?

**Hint:** Can you express any linear operation on one qubit in terms of the identity (no error), a bit-flip, a phase-flip, and a simultaneous bit and phase flip?

3. Suppose  $H$  is a linear subgroup of  $\mathbb{Z}_2^n$ . Verify that the Hadamard transform (the Fourier transform over  $\mathbb{Z}_2^n$ ) maps a uniform superposition over any coset of  $H$  onto a superposition over the perp subgroup  $H^\perp = \{g : g \cdot h = 0, \forall h \in H\}$ . Here,  $x \cdot y$  denotes the scalar product  $\sum_{i=1}^n x_i y_i \pmod{2}$ .