Physics becomes the computer

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Computing Beyond Silicon Summer School
Physics becomes the computer

Nature as Computer

Incorporating comp-universality at small and large scales

Spatial Worlds

Emulating Physics

Physical concepts enter CS and

Finite-state, locality, invertibility, and conservation laws

Spatial Computers

Architectures and algorithms for large-scale spatial computations
Looking at nature as a computer
Looking at computation as physics
Looking at nature as a computer
Introduction

As we zoom in on a digital image,
As we zoom in on a digital image, we begin to notice that there isn’t an infinite amount of resolution:
Introduction

As we zoom in on a digital image, we begin to notice that there isn’t an infinite amount of resolution:  
*We begin to see the pixels.*
Something similar happens in nature. A box full of particles doesn’t have an infinite number of possible configurations.
Something similar happens in nature. A box full of particles doesn't have an infinite number of different configurations: the number of distinct configurations is finite.
Similarly, the rate at which a finite system can transition from one distinct state to another is also finite.
Introduction

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Similarly, the rate at which a finite system can transition from one distinct state to another is also finite. Thus a finite physical system is much like a computer.
Introduction

Physics studies macro properties of finite information systems.

Basic quantities such as Entropy and Energy are informational:

\[ \text{Kinetic Energy} = \text{Max Ops} \]
\[ \text{Entropy} = \text{Max Info} \]

\[ S_{TL} = \sigma \rho \]
Introduction

- Physics studies macro properties of finite information systems.
- Basic quantities such as Entropy and Energy are informational.
- $K_{\text{ops}}^{\text{MAX}} = N_{\text{ops}}^{\text{MAX}}$  
- $S_{\text{MAX}} = I_{\text{MAX}}$

(1996, with Levitin)
In this talk...

Review:
• info (Entropy) in physics

Discuss:
• statistical description of computation ($\leftarrow$ QM)
• energy and action in comp
• intro (Entropy) in physics
• what does QM add?

In this talk...
What is Info?

• number of bits system can hold, given its constraints
• system with $2^n$ possible states can represent $n$ bits
• focus on classical info: system with $2^n$ possible states can represent $n$ bits
• ordinary macro quantities
  « when QM is invisible
  substitute micro dynamics
  « survives in macro limit
  « have classical info interp

$$\begin{align*}
\text{Info} & = - \sum p_i \log p_i \\
& = - \sum_i \Omega \log \Omega \\
& = - \sum_i \Omega \log \Omega
\end{align*}$$

% equally probable states,
What is Entropy?

- Formally a parameter in thermo (irreversibility)
- Boltzmann and Gibbs understood as counting
- Mixing neat → mess
- Mixing mess → mess
- Entropy is log of states that fit with constraints
Classical Entropy

- For particles in a box, can introduce some coarseness
- This allows relative probabilities to be calculated
- Also do the same thing for momentum

For particles in a box,
Infinite Entropy?

Thermo of EM radiation in cavity led to QM

General state is a superposition of waves with integer num peaks

Any amplitude, can put unit of energy into any wave (infinite info!)

Planck proposed $E = nh\nu$

EM radiation in a cavity (periodic boundaries)

(infinite info!)
Looking at nature as a computer

- With QM, every finite system has finite state
- Dynamics of finite systems is familiar
- Develop QM from computer viewpoint
- Begin by discussing computer logic in statistical situations
Looking at computation as physics

• With QM, every finite system has finite state
  • Dynamics of finite state systems is familiar
  • Develop QM from computer viewpoint
  • Begin by discussing computer logic in statistical situations

Looking at computation as physics
Statistical Dynamics

• To give a complete dynamics, we say what happens to each state in a fixed time.

• Weighted sum of states (superposition) describes an ensemble.

• Probability of initial state applies to corresponding final state.

Statistical Dynamics
• Better to use square roots of probabilities (amplitudes)
• Evolution preserves vector length
• Lets us analyze system in other bases

Statistical Dynamics

\[ \langle 01|p\rangle + \langle 11|q\rangle + \langle 10|r\rangle + \langle 00|s\rangle \]
\[ \iff \]
\[ \langle 11|p\rangle + \langle 01|q\rangle + \langle 10|r\rangle + \langle 00|s\rangle \]

\[ \langle 01\rangle = \langle 11|_{\text{XOR}} \]
\[ \langle 11\rangle = \langle 01|_{\text{XOR}} \]
\[ \langle 10\rangle = \langle 10|_{\text{XOR}} \]
\[ \langle 00\rangle = \langle 00|_{\text{XOR}} \]

\[ A \oplus B \]

\[ A \]
Energy Basis

Suppose \( \mathcal{U} \) represents one clock period of a reversible computer.

This state in orbit has equal prob for any config.

Time evolution leaves this state unchanged.

Add together all configs in orbit. This state has equal prob for any config.

\[
\langle 0 \mathcal{E}\rangle = \langle 0 \mathcal{E}\rangle^2 \mathcal{U}
\]

\[
\langle 0 \mathcal{E}\rangle = \langle 0 \mathcal{E}\rangle^2 \mathcal{U}
\]

\[
\langle x^0 \rangle + \cdots + \langle x^\pi \rangle + \langle x^1 \rangle \frac{N^\wedge}{I} = \langle 0 \mathcal{E}\rangle^2 \mathcal{U}
\]

\[
\langle x^{1-N} \rangle + \cdots + \langle 0 \rangle + \langle x^0 \rangle \frac{N^\wedge}{I} = \langle 0 \mathcal{E}\rangle
\]
**Energy Basis**

- Example: Suppose a computer only has one bit.
- Form a new 2-state basis by adding and subtracting configurations and raising to the power of 1/2.

\[ \langle 1 | = \langle 0 |^2 \sqrt{\mathcal{U}} \quad \langle 0 | = \langle 0 |^2 \sqrt{\mathcal{U}} \]

\[ \frac{\langle 1 | - \langle 0 |}{\langle 1 | + \langle 0 |} = \langle 1 | \sqrt{\mathcal{U}} \quad \frac{\langle 1 | + \langle 0 |}{\langle 1 | - \langle 0 |} = \langle 0 | \sqrt{\mathcal{U}} \]
Energy Basis

In general: use complex amplitudes to form new orthogonal basis

- $|a\rangle$ is like a row vector
- $\langle a|$ is like a column vector of components
- $\langle a| \otimes \langle a|$ is like a column vector of complex conjugates

$$
\begin{align*}
\langle 0| & \leftarrow \langle 1^{-N}| \leftarrow \cdots \leftarrow \langle 1| \leftarrow \langle 0| : \sum_{m}^{N} E^m = \\
\langle u \mathbb{E} | & \mathbb{E} \langle u \mathbb{E} | \sum_{m}^{N} E^m = \\
\langle 1+| & \mathbb{E} \langle 1+| \sum_{m}^{N} E^m = \langle u \mathbb{E} |^2 \mathbb{E} \\
\langle u| & \mathbb{E} \langle u| \sum_{m}^{N} E^m = \langle u \mathbb{E} |
\end{align*}
$$

Energy Basis
Energy Basis

Energy Basis is Fourier Transform of config basis

Energy Basis

\[ \langle 0 \rangle \leftarrow \langle 1 \rangle \leftarrow \cdots \leftarrow \langle N \rangle \leftarrow \langle 0 \rangle \times \frac{1}{u} \times \frac{N}{u} = \nu \]

For a cycle:

\[ \langle u \rangle \cdot \langle N/n \rangle = \nu \]

We will call \( u \) the Energy

of the state \( \langle \nu \rangle \), where \( \nu = \frac{1}{v} \times \frac{N}{n} \) cycles with a frequency

\[ \langle u \rangle \leftarrow \langle 1 \rangle \leftarrow \cdots \leftarrow \langle N \rangle \leftarrow \langle 0 \rangle \times \frac{1}{u} \times \frac{N}{u} = \nu \]
Energy Basis

- Interpret coefficients in energy basis as probs.
- Energy of any state is independent of time.
- $|X_n\rangle$ is composed of equally spaced energies, $E_n = nh\nu_1$.
- $E = \frac{\hbar \nu_1}{2}$ or $\nu = 2E/\hbar$.
- $\nu = \frac{1}{\hbar \eta}$.

Energy levels are

|\nu| |\nu| |\nu| |\nu|

For any state $|X\rangle$, coefficients are

$\langle \psi | \nu | \psi \rangle = \nu$

$E = \frac{\hbar \nu_1}{2}$, or $\nu = 2E/\hbar$.

Energy basis as probs

$\langle 0 | X | 0 \rangle \leftarrow \langle 1-N | X | 1-N \rangle \leftarrow \cdots \leftarrow \langle 1 | X | 1 \rangle \leftarrow \langle 0 | X | 0 \rangle$.

Energy Basis
What is energy?

Energy is extensive!

as bit changes (i.e.,
should count changes
reversible rules
one spot at a time for
CA lattice can change
configurations
rate of change of

ν = 2E/h, so energy is

\[ \langle 0 \rangle x \leftarrow \langle 1^{-N} \rangle x \leftarrow \ldots \leftarrow \langle 1 \rangle x \leftarrow \langle 0 \rangle x \]
What is Energy?

- Conservation Law: number of ones constant
- Constrains number of spots that can change
- Focus on energy of the lattice update period $\tau$
- If each particle is assigned an energy $\hbar$ and $\frac{\Delta \mathcal{E}}{\mu \nu l} = 2\mathcal{E}/\hbar$
- Max change is still $\frac{2\mathcal{E}}{\hbar}$
- $\nu = 2\mathcal{E}/\hbar\nu$
- $\nu = \nu_0$ particle energy
- $\nu = \nu_0$ num particles = $\mathcal{W}$

$\nu = \nu_0 + 1$
$\nu = \nu_0$
$\nu = \nu_0$
What is Action?

- Action is amount of evolution (total ops for ideal computation)
- Number of comp events in rest frame is rel scalar
- Comp energy must transform like rel energy:
  \[ \frac{2E'}{\hbar} = \frac{2(E - px)}{\hbar} \]
- If \( x/t = c \), then \( E = cp \) so \( E = c \frac{2E'}{\hbar} \)

moving frame

rest frame

rest frame
• QM allows some new kinds of operations
• Any invertible evolution which preserves vector length is okay
• Probabilities can come and go!
• Only need to add extra single-bit operations
• QM comp is special case:

What does QM add?
\[\text{XOR} + \text{NOT} \leq \text{universal}\]

\[\begin{aligned}
\hat{\mathcal{N}} &= \theta \mathcal{N} : A / B = \theta \\
\hat{\mathcal{N}} &= \theta \mathcal{N} : A / B = \theta \\
\langle 1 | \theta \cos + \langle 0 | \theta \sin = \langle 1 | \theta \mathcal{N} \\
\langle 1 | \theta \sin - \langle 0 | \theta \cos = \langle 0 | \theta \mathcal{N}
\end{aligned}\]
The XOR gates are universal!
\[ \langle 1 | \theta \cos + \langle 0 | \theta \sin = \langle 1 |^\theta \wedge \]
\[ \langle 1 | \theta \sin - \langle 0 | \theta \cos = \langle 0 |^\theta \wedge \]

\[ \text{are universal!} \]

\[ \text{NOT} \bigwedge + \text{XOR} \]
What does QM add?

- No new kinds of computations; at most reduces effort required.
- Distinction is basis dependent.
- Speedup is exponential.

Fundamental Q: If speedup is exponential, then distinction is real!
What does this mean?

Quantum:
\[
\langle 1| \frac{2}{q^- - p^-} + \langle 0| \frac{2}{q^+ + p^+} \quad \overset{\text{ION}}{\longrightarrow} \quad \langle 1| q^+ + \langle 0| p^-
\]

Classical:
\[
\langle 1| p^- - \langle 0| q^- \quad \overset{\text{ION}}{\longleftrightarrow} \quad \langle 1| q^+ + \langle 0| p^-
\]
What does this mean?

Quantum:

\[
\langle I | \frac{z^\dagger}{q^\dagger - p^\dagger} + \langle 0 | \frac{z^\dagger}{q^\dagger + p^\dagger} \quad \text{ION}^\dagger \quad \langle I | q^\dagger + \langle 0 | p^\dagger
\]

Classical:

\[
\langle I | p^\dagger - \langle 0 | q^\dagger \quad \text{ION} \quad \langle I | q^\dagger + \langle 0 | p^\dagger
\]
Conclusions

• On a large scale, often can't tell if micro finite-state is QM or CM
• Entropy, Energy and Action all have compelling meaning: others must
  state is QM or CM
• Signiﬁcant for comp and for physics
  On a large scale, often can't tell if micro finite-state is QM or CM
• Significant for comp
  for more information, see http://www.ai.mit.edu/people/nhm/looking-at-nature.pdf