THE BRAIN

I. INTRODUCTION

A. Human Brain

1) mass $\sim 1 - 2\,\text{kg}$ in mature adult
  a) about 2% of body weight
    i) uses 20% of oxygen, 25% of glucose, 15% of blood flow
  b) mass at birth about 20% of final value
    i) mass increase due to growth of axons, dendrites, synapses, myelin sheaths

B. Cortex

1) size of cortex separates humans from other species
  a) area: $5\,\text{cm}^2$ for rat, $5 \times 10^2\,\text{cm}^2$ for chimp, $2 \times 10^3\,\text{cm}^2$ for human
    i) extra area in human cortex obtained by folding
    ii) thickness of cortex $\sim 0.3\,\text{cm}$
  b) $\gtrsim 3 \times 10^{10}$ neurons in human cortex
    i) mammalian cortex has $\sim 1.5 \times 10^7$ neurons per cm^{-2}
  c) $\gtrsim 10^{14}$ synapses in human cortex
    i) $\gtrsim 10^3$ synapses per neuron
2) human genome does not carry detailed wiring diagram for cortex
  a) its information content is far too small
    i) wiring diagram would require $\gtrsim 10^{14}$ bits of information
  b) genome carries about 5×10^9 bits of information
    i) human genome is about one meter of DNA
    ii) 4 types of base pairs
    iii) separation of $4 \times 10^{-8}\,\text{cm}$ between base pairs
    iv) much of genome may be nonsense
3) cortex develops in response to external stimuli
  a) molecular markers involved in initial wiring
  b) refinements due to activity
  c) number of synapses pared back during development

C. Explosion In Size Of Cortex Due To Limited Genetic Instructions

1) comparison of evolution of genome and cortex
  a) genome length: 4 cm for fruit fly, 40 cm for chicken, 1 m for mouse, 1 m for human

b) number of neurons: 10^5 for fruit fly, 5×10^6 for mouse, 10^{11} for human

2) rapid evolution during past 3×10^6 yr

II. FUNCTION

A. Neuron

1) components: cell body, axon, dendrites

2) nominal dimensions for pyramidal cell in cortex

a) cell body: blob with $r \sim 20\,\mu$m

b) axon: cylinder with $r \sim 1\,\mu$m and $l \sim 1\,$cm

c) total surface area about $6 \times 10^{-4}\,\text{cm}^2$

i) dominated by axon

d) total volume about $10^{-8}\,\text{cm}^3$

i) comparable contributions from axon and cell body

2) electrical properties

a) axons are output devices

i) actively propagate signals

ii) contain repeater stations

b) dendrites are input devices

i) electrically passive

ii) some may produce spikes

3) synapses

a) connect axons to dendrites

i) signals transmitted chemically across synapses

ii) synaptic space $\approx 2 \times 10^{-6}$ cm

iii) time delay $\sim 0.1\,$ms due to diffusion

iv) to achieve post synaptic threshold may take much longer

b) can be excitatory or inhibitory

i) excitatory: glutamate transmitter opens Na channels (MSG)

ii) inhibitory: gaba transmitter opens Cl or K channels

iii) most common neurotransmitters in cortex

iv) amino acids

v) inhibitory synapses usually more proximal to cell body

B. Action Potentials

1) neuron sums inputs

a) strength related to distance of synapse on dendrite from cell body

b) sum determines whether firing occurs

2) spikes initiated on axon close to cell body

a) can travel in both directions along axon

3) pulses are quantized, all the same

a) pulse length $\sim 1\,\mathrm{ms}$

b) pulse strength $\Delta V \sim 10^2\,\mathrm{mV}$

c) length and strength determined by kinetics of ion channels

4) signal strength coded in firing rate ν

a) at rest: $0 \lesssim \nu \lesssim 50\,\mathrm{Hz}$, typically $\nu \sim 5\,\mathrm{Hz}$

b) excited: $2 \lesssim \nu \lesssim 200\,\mathrm{Hz}$, typically $\nu \sim 50\,\mathrm{Hz}$

i) limited to $\nu \lesssim 10^3\,\mathrm{Hz}$ by refractory period of ion channels

5) propagation speed

a) depends on axon diameter and myelination

i) $v \sim 5\,\mathrm{m\,s^{-1}}$ typical value for brain

ii) up to $v \sim 100\,\mathrm{m\,s^{-1}}$ in spinal cord

C. Axon Modeled As Coaxial Cable

1) parameters

a) radius a, membrane thickness t, length, l

i) typical values: $a \sim \mu\mathrm{m}$, $t \sim 7 \times 10^{-7}\,\mathrm{cm}$, $l \sim 1\,\mathrm{cm}$

b) longitudinal resistance, $R_a = \rho_a/\pi a^2 l$

i) salt solution, $\rho_a \approx 30\,\mathrm{ohm\,cm}$

i) $R_a \sim 1 \times 10^9(\,\mu\mathrm{m}/a)^2(l/\,\mathrm{cm})\,\mathrm{ohm}$

c) membrane capacitance, $C_m = 2\pi\epsilon_0 K a l/t$

i) dielectric constant of lipid membrane, $K \approx 6$

ii) $C_m \approx 7 \times 10^{-4}(a/\,\mu\mathrm{m})(l/\,\mathrm{cm})\,\mu\mathrm{F}$

iii) $C_m/A \approx 1\,\mu\mathrm{F\,cm^{-2}}$

d) membrane resistance, $R_m = \rho_m t/(2\pi a l)$

i) $\rho_m \approx 1.5 \times 10^9\,\mathrm{ohm\,cm}$

ii) $\rho_m t \approx 10^3\,\mathrm{ohm\,cm^2}$

iii) $R_m \approx 2 \times 10^6(\,\mu\mathrm{m}/a)(\,\mathrm{cm}/l)\,\mathrm{ohm}$

e) inductance negligible

D. Action Potential Propagation Along Unmyelinated Neuron

1) longitudinal diffusion

a) neglect current through membrane

$$\frac{\partial V}{\partial t} = -\frac{l}{C_m}\frac{\partial I}{\partial z}$$

$$\frac{\partial V}{\partial z} = -\frac{R_a}{l} I$$

b) diffusion equation

$$\frac{\partial V}{\partial t} = \frac{l^2}{R_a C_m}\frac{\partial^2 V}{\partial z^2}$$

ii) diffusion constant, $D \equiv l^2/R_a C_m \sim 1.5(a/\,\mu\mathrm{m})\,\mathrm{cm}^2\,\mathrm{s}^{-1}$

c) $\lambda \sim (D\Delta t)^{1/2}$, spreading length for pulse of duration Δt

i) $\lambda \sim 4 \times 10^{-2}(a/\,\mathrm{cm})^{1/2}(\Delta t/\,\mathrm{ms})^{1/2}\,\mathrm{cm}$

d) propagation speed along axon

$$v \sim \frac{\lambda}{\Delta t} \sim 40\left(\frac{a}{\mu\mathrm{m}}\right)^{1/2}\ \mathrm{cm\,s}^{-1}$$

i) evaluated for $\Delta t \approx 1\,\mathrm{ms}$

2) leakage through membrane

a) clamp voltage of axoplasm

$$\frac{\partial V}{\partial t} = -\frac{V}{R_m C_m}$$

b) voltage decays exponentially with time constant $\tau = R_m C_m$

i) $\tau \sim 1.5 \times 10^{-3}\,\mathrm{s}$

3) combined equation reads

$$\frac{\partial V}{\partial t} + \frac{V}{R_m C_m} = \frac{l^2}{R_a C_m}\frac{\partial^2 V}{\partial z^2}$$

a) impulse regeneration not included in equation

E. Action Potential Propagation Along Myelinated Neuron

1) myelin sheath decreases C_m

a) $10 - 15$ wraps of myelin sheath per micron diameter of axon

i) like paper towels on cardboard roller

b) $C_m \sim 3 \times 10^{-5}(l/\,\mathrm{cm})\,\mu\mathrm{F}$

i) note C_m independent of a

2) cross membrane currents restricted to nodes of Ranvier

a) separated by a few mm

a) size a few μm

3) myelination increases propagation speed at fixed size

a) $\lambda \sim 2 \times 10^{-1}(a/\,\mu\text{m})\,\text{cm}$

b) $v \sim 2(a/\,\mu\text{m})\,\text{m}\,\text{s}^{-1}$

i) note: $v \propto a$

F. Power Requirements

1) $P = C(\Delta V)^2\nu/2$

a) $P \sim 3 \times 10^{-12}\nu$ watt for our canonical neuron

b) $\sim 10^{11}$ neurons firing at $\nu \sim 10\,\text{Hz}$ yields a total power $\sim 3\,$watt

c) based on unmyelinated axons

i) myelination decreases power usage

ii) do small, unmyelinated axons use most of electic power?

iii) could dendrites use significant power?

2) total power used by brain $\sim 20\,$watt

a) how much for ion pumps?

b) how much for axon transport?

i) molecular motors

3) experimental indications

a) ion transport is major part of metabolism

i) barbiturate anesthesia producing isoelectric EEG reduces metabolism to 40% of normal value

ii) inhibiting Na-K pump using ouabain reduces metabolism to 20% of normal value

III. INFORMATION INPUT AND STORAGE

A. Eye

1) retina is 2.4 cm behind cornea, pupil size $0.2 \lesssim p \lesssim 0.4\,\text{cm}$

a) 10^7 cones

i) maximum density in center of fovea, $1.5 \times 10^7\,\text{cm}^{-2}$

b) 10^8 rods

i) maximum density 20° from center of fovea, $1.6 \times 10^7\,\text{cm}^{-2}$

ii) can detect single photon

c) cone acuity 10 times rod acuity, less convergence

   d) cone sensitivity at fovea 10 times smaller than rod sensitivity at 20°

2) resolution of eye at fovea $\Delta\theta \sim 5 \times 10^{-4}$ rad ~ 2 arc minutes
   a) density of cones matches diffraction limited resolution of eye
      i) diffraction limit: $\Delta\theta \sim \lambda/p \sim 2.5 \times 10^{-4}$
      ii) cone spacing: $\Delta\theta \sim 10^{-4}$
3) input from visual receptors funnels into $\sim 10^6$ neurons in optic nerve
   a) optic nerve can transmit $\sim 10^7$ bits per second
4) auditory nerve has $\sim 3 \times 10^4$ neurons
   a) auditory bandwidth is $\sim 2 \times 10^4$ Hz

B. Television

1) standard TV channel uses $\Delta\nu \approx 6$ MHz in range $50 - 1,000$ MHz
   a) only $\Delta\nu \approx 4$ MHz for picture
   b) 2.11×10^5 picture elements
      i) 495 horizontal lines
   c) raster scans at 60 frames per second
      i) more than 40 frames per second needed to avoid flicker
   d) 10^7 elements per second
2) angular scale of picture element
   a) 50 cm $\times$ 50 cm screen
      i) element size, $\Delta x \approx \Delta y \approx 0.1$ cm
   b) viewed at distance of $d \approx 3$ m
   c) $\Delta\theta \sim 3 \times 10^{-4}$ rad
3) match of visual input to TV
   a) $\Delta\nu \approx 4$ MHz matches capacity of optic nerve
      i) $\sim 10^6$ neurons firing at $\nu \sim 10$ Hz
   b) $\Delta\theta \sim 5 \times 10^{-4}$ rad matches angular separation of picture elements
4) aliens might wonder which came first, the TV or the eye

C. Memory

1) Hebb proposed that information is stored in strength of synaptic connections
   a) suppose that there are there N discernible levels of synaptic strength
   b) human brain might be able to store $\gtrsim 10^{14} \ln_2(N)$ bits
      i) every bit in one full year of viewing TV